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Generalized solution and effectiveness for concentric tube heat exchangers

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Abstract—In this paper, general solutions are obtained for the steady-state temperature of heat exchanging fluids along the length of a concentric tube heat exchanger. Heat exchanger effectiveness is also obtained in terms of the dimensionless exit temperature. Governing equations in non-dimensional differential form for the inner and outer fluid streams representing non-adiabatic conditions at the outside surface of the outer tube are solved analytically. Both counter-flow and parallel-flow cases are considered. Expressions for heat transfer to or from the outside are obtained. Exact agreement with the *NTU* method for adiabatic conditions at the outside surface and also the heat balance analysis provide validation of the generalized solution.

INTRODUCTION

THIS PAPER presents a theoretical analysis of a concentric tube heat exchanger under steady-state conditions. A general case where the non-adiabatic condition at the outside surface of the outer pipe exists, is considered. In this case, the fluid in the annular space exchanges heat with the fluid in the inner pipe as well as outside. The analysis allows the prediction of the temperatures of the fluid streams along the length of such heat exchangers where the commonly used *LMTD* method [1] and/or *NTU* method [2] do not apply. Governing equations in dimensionless form are derived for the fluid streams and are solved analytically. The inlet temperatures of the fluid streams provide the required boundary conditions. An expression for the amount of heat transferred across the non-adiabatic outside surface is also developed. The counter-flow as well as the parallel-flow type of concentric tube heat exchanger is analysed. Using the present solution, the effect of various dimensionless parameters on the performance of a concentric tube heat exchanger is shown. These solutions, for adiabatic conditions at the outside surface, are compared with the *NTU* method [2].

THEORETICAL ANALYSIS

General solutions for a concentric tube heat exchanger, counter-flow (Fig. 1(a)) and parallel-flow (Fig. 1(b)), are obtained by analytically solving the governing differential equations. These equations are derived with the following assumptions:

(1) mass flow rates m_1 and m_2 of the heat exchanging fluids are constant;

(2) inlet fluid temperatures T_{1i} and T_{2i} are constant;

(3) temperature of the outside environment (T_o) is constant;

(4) fluid properties are constant;

(5) the overall heat transfer coefficients remain constant;

(6) T_1 and T_2 represent average fluid temperatures at any section of the inner and annular fluid streams, respectively.

Mathematical model—counter-flow heat exchanger

Energy balance over a differential element of the inner and annular fluid in a counter-flow heat exchanger results in the following differential equations:

inner pipe

$$m_1 c_1 \frac{dT_1}{da_1} + U_1(T_2 - T_1) = 0; \qquad (1)$$

annulus

$$m_2 c_2 \frac{dT_2}{da_1} + U_1(T_2 - T_1) + U_o(T_2 - T_o)\frac{da_o}{da_1} = 0; \qquad (2)$$

boundary conditions

$$\text{at } a_1 = A_1, \quad T_1(A_1) = T_{1i} \qquad (3a)$$

$$\text{at } a_1 = 0, \qquad T_2(0) = T_{2i} \qquad (3b)$$

where da_1 and da_o represent the differential surface areas for the inner and outer pipe, respectively.

These governing equations are transformed into the following dimensionless form:

inner pipe

$$\dot{\theta}_1 - N\theta_1 + N\theta_2 = 0; \qquad (4)$$

NOMENCLATURE

a_o, a_1 surface area of pipe at any section [m^2]
A_o, A_1 total surface area of the pipe [m^2]
A dimensionless surface area of the inner pipe at any section, a_1/A_1, $0 \leqslant A \leqslant 1$
c specific heat of fluid [$J\,kg^{-1}\,K^{-1}$]
C heat capacity ratio, $m_1c_1/(m_2c_2)$ [dimensionless]
D diameter of pipe [m]
E, F constants
K dimensionless parameter, $U_oa_o/(U_1a_1) = U_oA_o/(U_1A_1)$
L length of pipe [m]
m mass flow rate [$kg\,s^{-1}$, $kg\,min^{-1}$]
N modified number of transfer units based on inner flow, $U_1A_1/(m_1c_1)$ [dimensionless]
NTU number of transfer units as defined in NTU method, $U_1A_1/(mc)_{min}$ [dimensionless]
Q heat flow rate [W]
T temperature [K, °C]
U overall heat transfer coefficient [$W\,m^{-2}\,K^{-1}$].

Greek symbols
β defined by equations (8c) and (26c)
γ defined by equations (8b) and (26b)
ε heat exchanger effectiveness
θ dimensionless temperature defined by equation (7a)
$\dot{\theta}$ first derivative of $\theta(A)$ with respect to $A = d\theta/dA$
$\ddot{\theta}$ second derivative of $\theta(A)$ with respect to $A = d^2\theta/dA^2$
λ_1, λ_2 defined by equations (9b) and (27b).

Subscripts
o outside or outer pipe
1 inner pipe or fluid in the inner pipe
2 fluid in the annular space
e exit
i inlet
max maximum
min minimum
ntu *NTU* method.

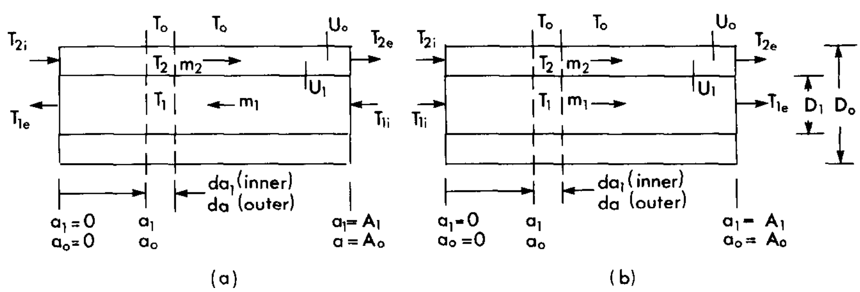

FIG. 1. Schematic diagram of a concentric tube heat exchanger: (a) counter-flow; (b) parallel-flow.

annulus

$$\dot{\theta}_2 + (1+K)NC\theta_2 - NC\theta_1 = KNC\theta_o; \quad (5)$$

boundary conditions

$$\text{at } A = 1, \quad \theta_1(A) = \theta_{1i} = 0 \quad (6a)$$

$$\text{at } A = 0, \quad \theta_2(A) = \theta_{2i} = 1 \quad (6b)$$

where the following definitions for the dimensionless terms are used:

dimensionless temperature,

$$\theta_n = \frac{T_n - T_{1i}}{T_{2i} - T_{1i}}, \quad n = 0, 1, 2 \quad (7a)$$

dimensionless area (inner pipe),

$$A = a_1/A_1, \quad 0 \leqslant A \leqslant 1 \quad (7b)$$

ratio of heat capacity rate,

$$C = (m_1c_1)/(m_2c_2) \quad (7c)$$

number of transfer unit (inner fluid),

$$N = U_1A_1/(m_1c_1) \quad (7d)$$

ratio of overall thermal resistances across the walls

$$K = U_oa_o/(U_1a_1) = U_oA_o/(U_1A_1). \quad (7e)$$

By combining equations (4) and (5) and eliminating the dimensionless temperature $\theta_2(A)$, the following equation in $\theta_1(a)$ is obtained:

$$\ddot{\theta}_1 + \gamma\dot{\theta}_1 + \beta\theta_1 = \beta\theta_o \quad (8a)$$

where

$$\gamma = N[(1+K)C - 1] \quad (8b)$$

and

$$\beta = -KCN^2 \quad (8c)$$

Analytical solution ($C \neq 1$ *if* $K = 0$)

The analytical solution for $\theta_1(A)$ from equation (8a) is given by

$$\theta_1(A) = E\exp(\lambda_1 A) + F\exp(\lambda_2 A) + \theta_o \quad (9a)$$

where

$$\lambda_1, \lambda_2 = 0.5[-\gamma \pm (\gamma^2 - 4\beta)^{1/2}]. \quad (9b)$$

The solution for $\theta_2(A)$ is obtained by substituting $\theta_1(A)$ and its first derivative $\dot{\theta}_1(A)$ from equation (9a) into equation (4) and is expressed as

$$\theta_2(A) = E(1-\lambda_1/N)\exp(\lambda_1 A) + F(1-\lambda_2/N)\exp(\lambda_2 A) + \theta_o. \quad (10)$$

Using the boundary conditions from equations (6), the constants E and F are obtained as

$$E = \frac{N\exp(\lambda_2) - \theta_o[N(\exp(\lambda_2)-1)+\lambda_2]}{(N-\lambda_1)\exp(\lambda_2)-(N-\lambda_2)\exp(\lambda_1)} \quad (11)$$

$$F = (-1)\frac{N\exp(\lambda_1) - \theta_o[N(\exp(\lambda_2)-1)+\lambda_1]}{(N-\lambda_1)\exp(\lambda_2)-(N-\lambda_2)\exp(\lambda_1)}. \quad (12)$$

Heat transferred to the environment

The amount of heat transferred to the environment through the non-adiabatic outside surface of the outer pipe is given by the following integral equation:

$$Q_o = \int_0^{A_o} U_o(T_2 - T_o)\,dA_o. \quad (13a)$$

Combining equations (7a)–(7c) and (10) into equation (13a) and integrating yields

$$Q_o = U_o A_o (T_{2i} - T_{1i})\{E(1-\lambda_1/N)[\exp(\lambda_1)-1]/\lambda_1 + F(1-\lambda_2/N)[\exp(\lambda_2)-1]/\lambda_2\}, \quad U_o \neq 0. \quad (13b)$$

Analytical solution ($C = 1$ and $K = 0$)

The above general solution for a concentric tube counter-flow heat exchanger applies to all values of C and K with the exception of $C = 1$ and $K = 0$. The following solutions are presented for this special case. Under these conditions, equation (8a) transforms into

$$\ddot{\theta}_1 = 0. \quad (14)$$

Therefore

$$\dot{\theta}_1 = E \quad (15)$$

and

$$\theta_1(a) = EA + F. \quad (16)$$

Substitution of equations (15) and (16) into equation (4), yields the following solution for $\theta_2(A)$:

$$\theta_2(A) = E(A - 1/N) + F. \quad (17)$$

By applying the boundary conditions in equation (6), the constants E and F are determined as

$$E = -N/(1+N); \quad \text{if } C = 1 \text{ and } K = 0 \quad (18)$$

$$F = N/(1+N); \quad \text{if } C = 1 \text{ and } K = 0. \quad (19)$$

The amount of heat transferred (Q_o) to the environment, in this case, is zero because $K = 0$ represents an adiabatic condition ($U_o = 0$) at the outside surface of the outer pipe.

Mathematical model—parallel-flow heat exchanger

A similar theoretical analysis as above for a parallel-flow heat exchanger with non-adiabatic conditions at the outside surface of the outer pipe, results in the following differential equations:

inner pipe

$$-m_1 c_1 \frac{dT_1}{da_1} + U_1(T_2 - T_1) = 0; \quad (20)$$

annulus

$$m_2 c_2 \frac{dT_2}{da_1} + U_1(T_2 - T_1) + U_o(T_2 - T_o)\frac{da_o}{da_1} = 0; \quad (21)$$

boundary conditions

$$\text{at } a_1 = 0, \quad T_1(0) = T_{1i} \quad (22a)$$

$$\text{at } a_1 = 0, \quad T_2(0) = T_{2i}. \quad (22b)$$

Using the dimensionless terms defined in equations (7), the above differential equations are transformed into the following dimensionless form:

inner pipe

$$\dot{\theta}_1 + N\theta_1 - N\theta_2 = 0; \quad (23)$$

annulus

$$\dot{\theta}_2 + (1+K)NC\theta_2 - NC\theta_1 = KNC\theta_o; \quad (24)$$

boundary conditions

$$A = 0, \quad \theta_1(A) = \theta_{1e} = 0 \quad (25a)$$

$$A = 0, \quad \theta_2(A) = \theta_{2i} = 1. \quad (25b)$$

Combining equations (23) and (24) and eliminating the dimensionless temperature θ_2, the following equation in θ_1 is obtained:

$$\ddot{\theta}_1 + \gamma\dot{\theta}_1 + \beta\theta_1 = \beta\theta_o \quad (26a)$$

where

$$\gamma = N[(1+K)C + 1] \quad (26b)$$

and

$$\beta = KCN^2. \quad (26c)$$

Analytical solution

The analytical solution for $\theta_1(A)$ from equation (26) is given by

$$\theta_1(A) = E\exp(\lambda_1 A) + F\exp(\lambda_2 A) + \theta_o \quad (27a)$$

where

$$\lambda_1, \lambda_2 = 0.5[-\gamma \pm (\gamma^2 - 4\beta)^{1/2}] \quad (27b)$$

and E and F are constants. Substitution of $\theta_1(A)$ from equation (27) and its derivative into equation (23) yields the following solution for $\theta_2(A)$:

$$\theta_2(A) = E(1+\lambda_1/N)\exp(\lambda_1 A) + F(1+\lambda_2/N)\exp(\lambda_2 A)+\theta_o. \quad (28)$$

The constants E and F are determined by applying the boundary conditions from equations (25) to equations (27) and (28), and found to be

$$E = \frac{N+\lambda_2\theta_2}{\lambda_1-\lambda_2}, \quad F = -\frac{N+\lambda_1\theta_o}{\lambda_1-\lambda_2}. \quad (29)$$

Heat transferred to the outside

The heat transferred through the outer surface of the outer pipe is given by

$$Q_o = \int_0^{A_o} U_o(T_2-T_o)\,\mathrm{d}A_o. \quad (30a)$$

Substitution of equations (7a)–(7c) and (28) into equation (30a) and integration yields the following expression:

$$Q_o = U_oA_o(T_{2i}-T_{1i})\{E(1+\lambda_1/N)[\exp(\lambda_1)-1]/\lambda_1 + F(1+\lambda_2/N)[\exp(\lambda_2)-1]/\lambda_2\}, \quad U_o \neq 0. \quad (30b)$$

RESULTS

Solutions for the dimensionless temperatures θ_1 and θ_2 of the fluids as a function of the dimensionless area A ($0 \leqslant A \leqslant 1$) are presented for counter-flow as well as parallel-flow heat exchangers in this section. Figures 2 and 3 show the variation of θ_1 and θ_2 as a function of A ($0 \leqslant A \leqslant 1$) for $N = 2$, $C = 0.5$ and $\theta_o = 0.5$ for non-adiabatic conditions ($K = 0.5$, 1.0 and 2.0) as well as the adiabatic condition ($K = 0$) at the outside surface. Significant variation in the performance of a double-pipe heat exchanger is observed due to non-adiabatic conditions. The influence of the outside temperature θ_o on θ_1 and θ_2 for $C = 0.5$, $N = 2$, $K = 0.5$ is shown in Figs. 4 and 5 and compared with those for adiabatic conditions ($K = 0$). Figures 6 and 7 present the results for θ_1 and θ_2 for $N = 2$, $\theta_o = 0.5$ and show the influence of variation

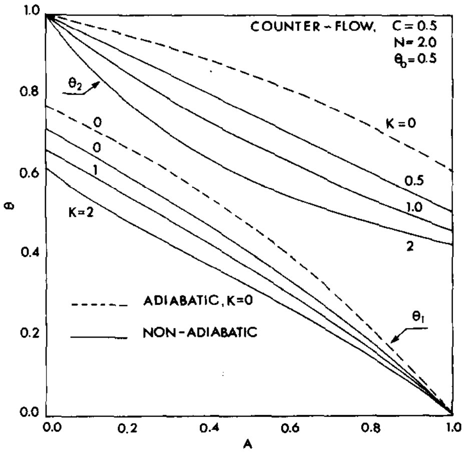

FIG. 2. Variation of θ_1 and θ_2 along the length of a counter-flow heat exchanger for $K = 0, 0.5, 1.0, 2.0$.

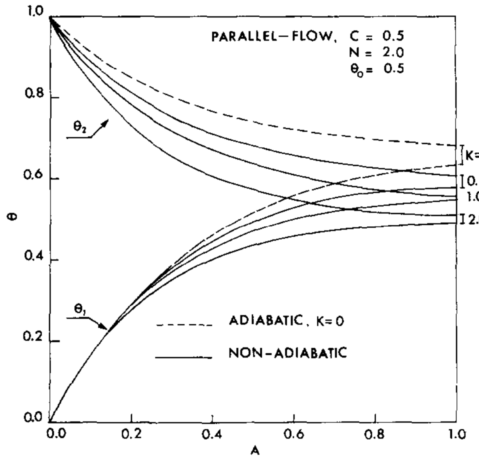

FIG. 3. Variation of θ_1 and θ_2 along the length of a parallel-flow heat exchanger for $K = 0, 0.5, 1.0, 2.0$.

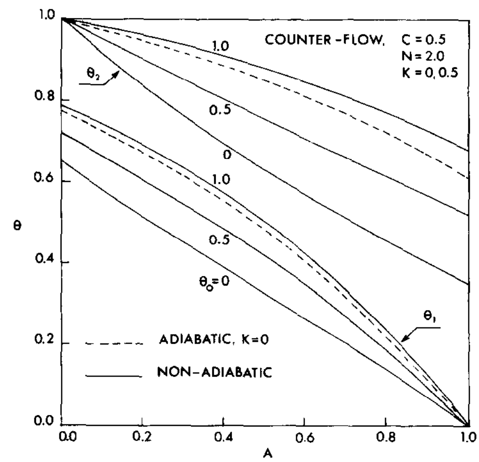

FIG. 4. Variation of θ_1 and θ_2 along the length of a counter-flow heat exchanger for $\theta_o = 0, 0.5, 1.0$.

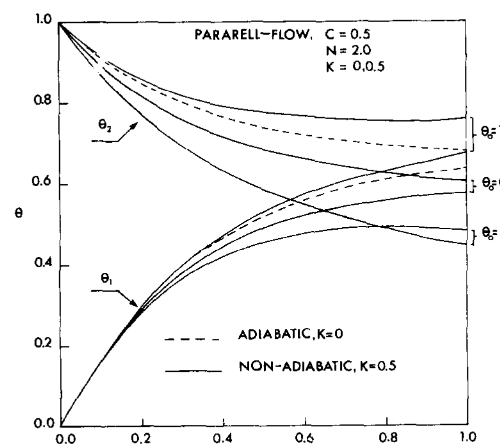

FIG. 5. Variation of θ_1 and θ_2 along the length of a parallel-flow heat exchanger for $\theta_o = 0, 0.5, 1.0$.

in the heat capacity ratio C. Results for adiabatic ($K = 0$) as well as non-adiabatic conditions ($K = 0.5$) are also shown in Figs. 6 and 7. The dimensionless temperatures θ_{1e} and θ_{2e} of the fluids at the exit of a

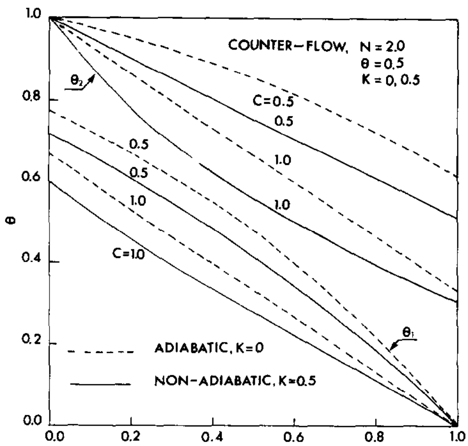

FIG. 6. Variation of θ_1 and θ_2 along the length of a counter-flow heat exchanger for $C = 0, 0.5, 1.0$.

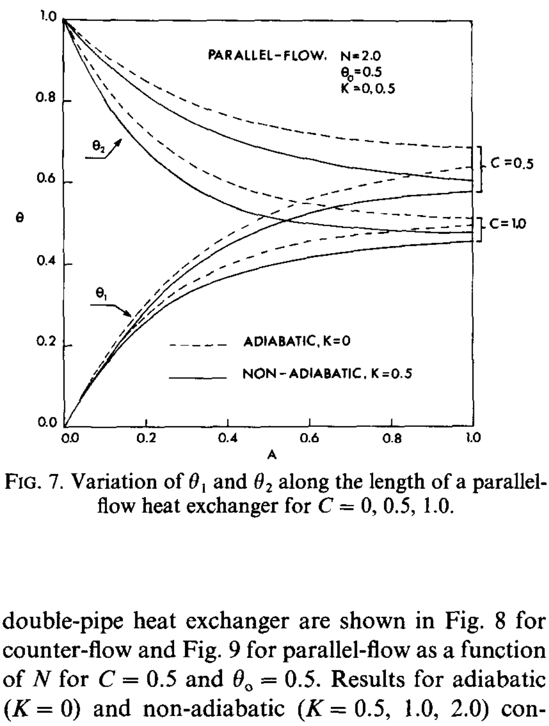

FIG. 7. Variation of θ_1 and θ_2 along the length of a parallel-flow heat exchanger for $C = 0, 0.5, 1.0$.

double-pipe heat exchanger are shown in Fig. 8 for counter-flow and Fig. 9 for parallel-flow as a function of N for $C = 0.5$ and $\theta_o = 0.5$. Results for adiabatic ($K = 0$) and non-adiabatic ($K = 0.5$, 1.0, 2.0) conditions are shown and compared (Figs. 8 and 9).

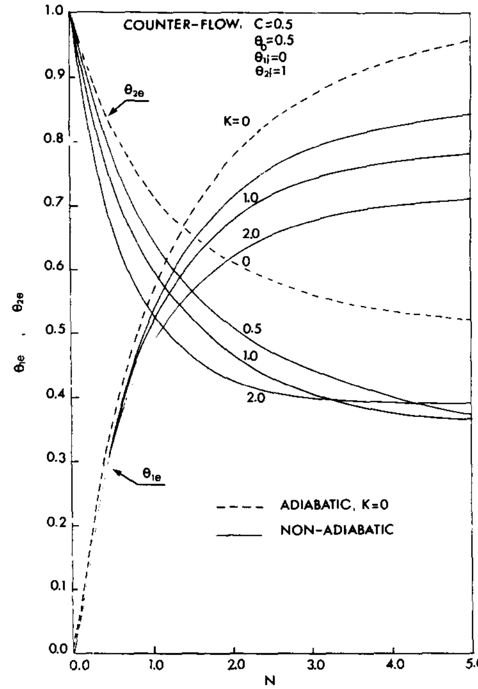

FIG. 8. Plot of θ_e–N for a counter-flow heat exchanger for $K = 0, 0.5, 1.0, 2.0$.

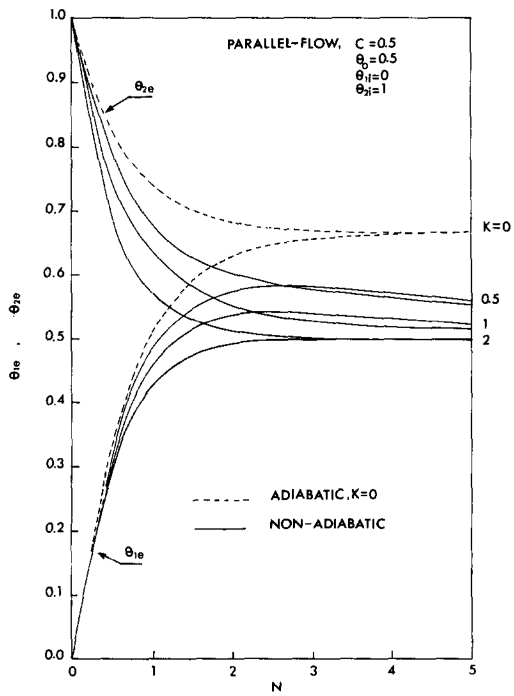

FIG. 9. Plot of θ_e–N for a parallel-flow heat exchanger for $K = 0, 0.5, 1.0, 2.0$.

HEAT EXCHANGER EFFECTIVENESS AND COMPARISON WITH *NTU* METHOD

The results of the present analysis permit determination of the heat exchanger effectiveness and are compared with the NTU method [1]. The heat exchanger effectiveness, ε, as defined in the NTU method [1] for $m_1c_1 > m_2c_2$ and $m_1c_1 < m_2c_2$ is used for its determination in general and comparison with the NTU method for adiabatic conditions ($K = 0$) at the outside surface of the heat exchanger.

For $m_1c_1 \leqslant m_2c_2$ ($C \leqslant 1$), ε for parallel-flow as well as counter-flow is defined as

$$\varepsilon = \frac{T_{1e} - T_{1i}}{T_{2i} - T_{1i}} = \varepsilon_{ntu} \tag{31}$$

which, with equation (7a) reduces to

$$\varepsilon = \theta_{1e} = \varepsilon_{ntu}(C_{ntu}, NTU) \tag{32a}$$

Table 1. Comparison of heat exchanger effectiveness for counter-flow heat exchanger

C	Present method with $K=0$		NTU method		
$C<1$	N	$\varepsilon=\theta_{1e}$	$C_{ntu}=C$	$NTU=N$	ε_{ntu}
0.5	0.5	0.362	0.5	0.5	0.362
	1.0	0.565		1.0	0.565
	2.0	0.775		2.0	0.775
	3.0	0.847		3.0	0.847
	4.0	0.927		4.0	0.927
	5.0	0.957		5.0	0.957
$C=1$	N	$\varepsilon=\theta_{1e}$ $\varepsilon=1-\theta_{2e}$	$C_{ntu}=C$ $C_{ntu}=1/C$	$NTU=N$ $NTU=NC$	ε_{ntu}
1.0	0.5	0.333	1.0	0.5	0.333
	1.0	0.500		1.0	0.500
	2.0	0.667		2.0	0.667
	3.0	0.750		3.0	0.750
	4.0	0.800		4.0	0.800
	5.0	0.833		5.0	0.833
$C>1$	N	$\varepsilon=1-\theta_{2e}$	$C_{ntu}=1/C$	$NTU=NC$	ε_{ntu}
2.0	0.5	0.565	0.5	1.0	0.565
	1.0	0.775		2.0	0.775
	2.0	0.927		4.0	0.927
	3.0	0.974		6.0	0.974
	4.0	0.991		8.0	0.991
	5.0	0.997		10.0	0.997

Table 2. Comparison of heat exchanger effectiveness for parallel-flow heat exchanger

C	Present method with $K=0$		NTU method		
$C<1$	N	$\varepsilon=\theta_{1e}$	$C_{ntu}=C$	$NTU=N$	ε_{ntu}
0.5	0.5	0.352	0.5	0.5	0.352
	1.0	0.518		1.0	0.518
	2.0	0.633		2.0	0.633
	3.0	0.659		3.0	0.659
	4.0	0.665		4.0	0.665
	5.0	0.666		5.0	0.666
$C=1$	N	$\varepsilon=\theta_{1e}$ $\varepsilon=1-\theta_{2e}$	$C_{ntu}=C$ $C_{ntu}=1/C$	$NTU=N$ $NTU=NC$	ε_{ntu}
1.0	0.5	0.316	1.0	0.5	0.316
	1.0	0.432		1.0	0.432
	2.0	0.491		2.0	0.491
	3.0	0.499		3.0	0.499
	4.0	0.500		4.0	0.500
	5.0	0.500		5.0	0.500
$C>1$	N	$\varepsilon=1-\theta_{2e}$	$C_{ntu}=1/C$	$NTU=NC$	ε_{ntu}
2.0	0.5	0.518	0.5	1.0	0.518
	1.0	0.633		2.0	0.633
	2.0	0.665		4.0	0.665
	3.0	0.667		6.0	0.667
	4.0	0.667		8.0	0.667
	5.0	0.667		10.0	0.667

where

$$C_{ntu} = \frac{(mc)_{\min}}{(mc)_{\max}} = C \tag{32b}$$

and

$$NTU = \frac{U_1 A_1}{m_1 c_1} = N. \tag{32c}$$

For $m_1c_1 \geqslant m_2c_2$ ($C \geqslant 1$), ε for parallel-flow as well as counter-flow is defined as

$$\varepsilon = \frac{T_{2i}-T_{2e}}{T_{2i}-T_{1i}} = 1 - \frac{T_{2e}-T_{1i}}{T_{2i}-T_{1i}} = \varepsilon_{ntu} \tag{33}$$

which, together with equation (7a) reduces to

$$\varepsilon = 1-\theta_{2e} = \varepsilon_{ntu}(C_{ntu}, NTU) \tag{34a}$$

where

$$C_{ntu} = \frac{(mc)_{\min}}{(mc)_{\max}} = \frac{1}{C} \tag{34b}$$

and

$$NTU = \frac{U_1 A_1}{m_2 c_2} = NC. \tag{34c}$$

Heat exchanger effectiveness, ε, calculated with the present method for various values of C and N for counter-flow as well as parallel-flow are listed in Tables 1 and 2, respectively. The corresponding values of C_{ntu}, NTU and heat exchanger effectiveness, ε_{ntu}, determined by the NTU method [1] are also tabulated. For $C = 0.5$ ($C < 1$), equations (32) are applicable and ε–N are found to be in exact agreement with ε_{ntu}–NTU obtained by the NTU method [1] for counter-flow (Table 1, Fig. 8) as well as parallel-flow (Table 2, Fig. 9). For $C = 1$, equations (32) and (34) both are applicable. As shown in Tables 1 and 2, results obtained with the present method are identical to those obtained by the NTU method. For $C = 2$ ($C > 1$), equations (34) are used to determine the effectiveness ε, NTU and C_{ntu}. The effectiveness, ε_{ntu}, obtained by the NTU method is in exact agreement (Tables 1 and 2).

HEAT BALANCE

Expressions for Q_o, heat transferred through the outside surface of the heat exchanger, are provided in equations (13b) and (30b). The heat gain Q_1 and Q_2 by the fluid in the inner pipe and the annulus, respectively, can be estimated with the following equations:

$$Q_1 = m_1c_1(T_{1e}-T_{1i}) \tag{35a}$$

$$Q_2 = m_2c_2(T_{2e}-T_{2i}). \tag{35b}$$

The algebraic sum of Q_o, Q_1 and Q_2

$$Q_o + Q_1 + Q_2 = 0 \tag{36}$$

would provide further validation of the present analysis.

A case study is presented here for the parameter

Table 3. Parameter values for case study

For inner tube	For annulus
$m_1c_1 = 1000.0\,\mathrm{J\,min^{-1}\,°C^{-1}}$	$m_2c_2 = 2000.0\,\mathrm{J\,min^{-1}\,°C^{-1}}$
$T_{1i} = 10.0°C$	$T_{2i} = 100.0°C$
$U_1 = 25.0\,\mathrm{W\,m^{-2}\,°C^{-1}}$	$U_o = 10.0\,\mathrm{W\,m^{-2}\,°C^{-1}}$
$L = 2.0\,\mathrm{m}$	$L = 2.0\,\mathrm{m}$
$D_1 = 0.05\,\mathrm{m}$	$D_o = 0.1\,\mathrm{m}$
$A_1 = 0.1\pi\,\mathrm{m^2}$	$A_o = 0.2\pi\,\mathrm{m^2}$

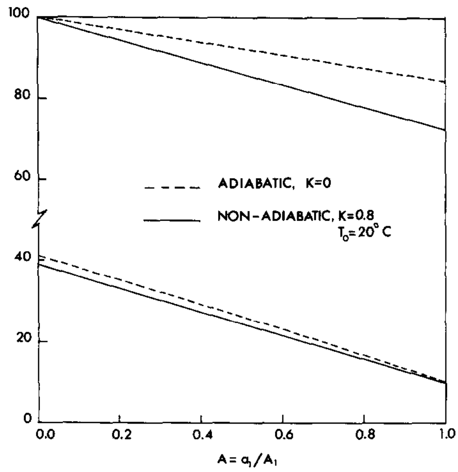

FIG. 10. Plot of T_1 and T_2 along the length of a counter-flow heat exchanger.

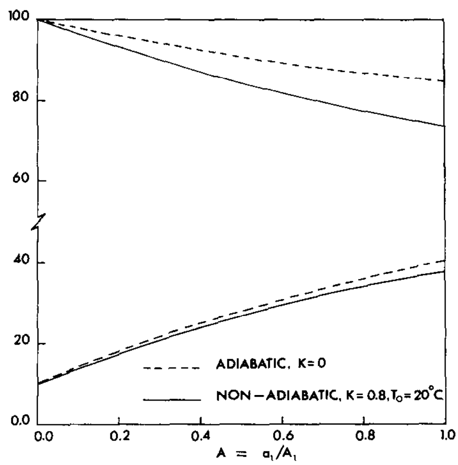

FIG. 11. Plot of T_1 and T_2 along the length of a parallel-flow heat exchanger.

values given in Table 3 and the results for counter-flow as well as parallel-flow are presented for $T_o = 20°C$ along with those for adiabatic conditions ($K = 0$ or $U_o = 0$) in Figs. 10 and 11, and Table 4. The exit temperatures T_{1e} and T_{2e} of the fluids, the effectiveness $\varepsilon = \theta_{1e}$ as well as the values of Q_o, Q_1 and Q_2 obtained by the present method are indicated. Verification of the heat balance equation (36) is also shown in Table 4. The exit temperatures T_{1e} and T_{2e}, and effectiveness, ε, are found to be in exact agreement with those obtained by the NTU method and provides

Table 4. Heat balance calculation—case study

	Counter-flow		Parallel-flow	
	$T_o = 20°C$	$U_o = 0$ Adiabatic	$T_o = 20°C$	$U_o = 0$ Adiabatic
m_1c_1†	1000.0	1000.0	1000.0	1000.0
m_2c_2†	2000.0	2000.0	2000.0	2000.0
U_1	25.0	25.0	25.0	25.0
U_o	10.0	0	10.0	0
A_1	0.1π	0.1π	0.1π	0.1π
A_o	0.2π	0.2π	0.2π	0.2π
T_{1i}	10.0	10.0	10.0	10.0
T_{2i}	100.0	100.0	100.0	100.0
T_o	20.0	20.0	20.0	20.0
Present method				
N	0.471	0.471	0.471	0.471
C	0.500	0.500	0.500	0.500
K	0.800	0	0.800	0
θ_o	0.111	0.111	0.111	0.111
$\theta_{1e} = \varepsilon$	0.322	0.347‡	0.311	0.338‡
θ_{2e}	0.700	0.827	0.708	0.831
T_{1e}	39.005	41.230‡	37.947	40.409‡
T_{2e}	73.032	84.385‡	73.701	84.796‡
Q_o	415.524	0	410.838	0
Q_1	483.416	520.494	465.782	506.809
Q_2	−898.940	−520.494	−876.620	−506.809
$\sum_0^2 Q_n$	0	0	0	0
NTU method				
ε_{ntu}	—	0.347	—	0.338
T_{1e}	—	41.230	—	40.409
T_{2e}	—	84.385	—	84.796

† In $\mathrm{J\,min^{-1}\,K^{-1}}$.
‡ Exact agreement with NTU method.

further validation of the present analysis. Results obtained by the generalized solution (Table 4) for non-adiabatic as well as adiabatic conditions are found to be in exact agreement with those obtained by the analytical method [3].

CONCLUSIONS

Generalized analytical solutions for the steady-state temperature of the heat exchanging fluids along the length of a counter-flow as well as a parallel-flow concentric tube heat exchanger are presented in terms of dimensionless parameters. These solutions are obtained, in general, for non-adiabatic conditions at the outer surface of such heat exchangers. However, if adiabatic conditions are assumed at the outside surface of the heat exchanger, these solutions are identical to those obtained by the NTU method. The present analysis also permits the determination of the heat exchanger effectiveness, as defined by the NTU method, from the dimensionless exit temperatures of the heat exchanging fluids. Expressions for heat transferred to the outside are also obtained. A case study is presented showing the heat balance for a heat exchanger for the non-adiabatic case as well as an exact agreement with the NTU method for adiabatic conditions at the outside surface of the heat exchanger.

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SOLUTION GENERALE ET EFFICACITE DES ECHANGEURS DE CHALEUR A TUBES CONCENTRIQUES

Résumé—On obtient des solutions générales pour la température stationnaire des fluides échangeant de la chaleur, tout le long d'un échangeur à tubes concentriques. On obtient aussi l'efficacité de l'échangeur en fonction de la température adimensionnelle de sortie. On résout analytiquement les équations aux dérivées partielles adimensionnelles pour les écoulements intérieur et externe, pour des conditions non adiabatiques à la surface extérieure du tube interne. On considère les cas de contre-courant et de co-courant. Sont obtenues des expressions pour le transfert thermique. Un accord exact avec la méthode NTU pour les conditions adiabatiques à la surface extérieure et aussi avec l'analyse du bilan thermique fournit la validation de la solution générale.

VERALLGEMEINERTES BERECHNUNGSVERFAHREN FÜR DOPPELROHR-WÄRMETAUSCHER

Zusammenfassung—In dieser Arbeit wird der stationäre Verlauf der Fluidtemperaturen entlang eines Doppelrohr-Wärmetauschers berechnet. Außerdem ergibt sich der Wärmetauscher-Wirkungsgrad als dimensionslose Austrittstemperatur. Für die inneren und äußeren Fluidströme werden die Bilanzgleichungen in differentieller dimensionsloser Form angegeben und analytisch gelöst, hierbei wird ein Wärmeaustausch mit der Umgebung zugelassen und sowohl Gleich- als auch Gegenstrom betrachtet. Für den Wärmetransport zur oder von der Umgebung werden Gleichungen angegeben. Das Ergebnis zeigt eine exakte Übereinstimmung mit der NTU-Methode für adiabate Bedingungen an der äußeren Oberfläche, ebenso bestätigt die Wärmebilanz die Gültigkeit der allgemeinen Lösung.

ОБОБЩЕННОЕ РЕШЕНИЕ И ЭФФЕКТИВНОСТЬ ДЛЯ КОНЦЕНТРИЧЕСКИХ ТРУБЧАТЫХ ТЕПЛООБМЕННИКОВ

Аннотация—Получены распределения для стационарной температуры жидкостей по длине концентрического трубчатого теплообменника. Эффективность выводится через безразмерную температуру на входе. Аналитически решаются определяющие уравнения в безразмерной форме для внутреннего и внешнего потоков жидкости при неадиабатических условиях на наружной поверхности внешней трубы. Рассматриваются случаи противотока и прямотока. Получены выражения для теплопереноса внутрь и наружу. Точное соответствие с данными, полученными методом количества единиц переноса для адиабатических условий на внешней поверхности, а также анализ теплового баланса обосновывают правомерность обобщенного решения.